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Effect of the Adverse Pressure Gradient on Vortex Breakdown

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The present work examines in detail the effect of the degree of divergence on the type and location of the vortex breakdown in swirling flows in tubes of various angles of divergence, compares the results with those predicted through the use of the analytical models proposed by Randall and Leibovich and by Mager, and illustrates the role played by the flow separation and reversal on the tube wall.

Nomenclature

a = normalized core area ($= \delta^2$)
A = normalized tube area
b_{ij} = a variable coefficient
D_o = minimum tube diameter ($= 2r_o$)
f_{ij} = functions of α, β, and a/A
I_{ij} = massflow and momentum deficiency integrals
K_{ij} = a variable coefficient
L = length of the diverging tube
m = const
M = massflow coefficient ($=$ massflow$/\pi\rho r_o{}^2 V_o'$)
p = static pressure
P = total pressure
r, z = radial and axial coordinates
r_o = minimum radius of the tube
r_b = tube wall radius
R_i = radial distance to the tip of a vane
Re = Reynolds number
S = swirl coefficient ($= \Gamma/r_o W_o$)
u, v, w = flow velocities in cylindrical coordinates
V_i = uniform velocity between two vanes
W = value of w outside the vortex core
$W_o, \bar{W}_o$ = maximum velocity and mean velocity at $z = 0$
α = ratio of axial velocities ($= w_o/W$)
β = a parameter
γ = half angle of divergence
Γ = circulation
δ = core radius
η = normalized radial distance ($= r/\delta$)
θ = vane angle
ν = kinematic viscosity
ρ = density
Ω = normalized circulation ($= \Gamma/\bar{W}_o D_o$)
$(\ )'$ = differentiation with respect to z

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Index categories: Viscous Nonboundary-Layer Flows; Hydrodynamics.

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Introduction

THIS paper is the sequel to the author's two previous publications.[1,2] The first of these emphasized the existence of a range of breakdown patterns, from a double-helix sheet to a highly axisymmetric bubble, and examined the swirl-angle distribution, variation of the axial velocity, and the characteristics of traveling vortex breakdowns. The second described the additional observations made and discussed the results in the context of existing theories. Here attention is directed to the effect of the adverse pressure gradient on the position of occurrence and the form of the vortex breakdown.

Despite numerous analytical and experimental attempts, which are aptly summarized by Hall,[3] the explanation of the mechanism giving rise to the vortex breakdown has withstood attack and remains a source of controversy. Consequently, little progress has been made towards the prediction of the effect of the occurrence or use of the vortex breakdown on delta wings, in suction tubes of pumps, in draft tubes of turbines, in flame holders and combustion chambers, in trailing vortices behind wings, etc., just to name a few of a variety of practical circumstances in which the vortex breakdown has favorable or unfavorable effects.

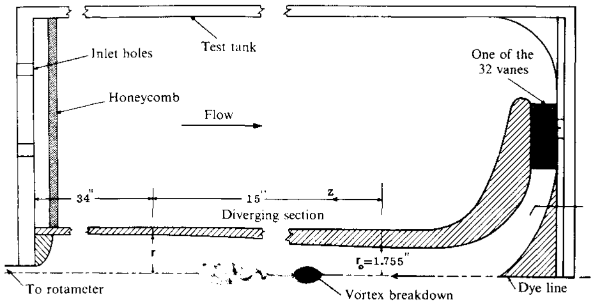

Fig. 1 Top half of the experimental apparatus.

A particularly important question and the most difficult aspect of the practical problems involving vortex breakdown is the prediction of the position of appearance of the breakdown. The difficulty arises primarily because of the fact that all of the parameters affecting the position of breakdown are not yet known. So far, it has been shown that[1-3] the occurrence of the breakdown and its location when it does occur depends, among other things, on the magnitude of the flow and the swirl, the external pressure gradient, the degree of divergence of the flow, and the initial upstream conditions. It is also strongly dependent upon all of the confinements placed upon it. Suffice it to say that the analytical treatment of the problem is extremely complicated partly because the phenomenon is sometimes quite sensitive to some of the variables and partly because the reciprocal action of the breakdown on the pressure-gradient boundary layers, and in short on the entire flowfield, cannot be readily accounted for. Thus, the exploration of the vortex-breakdown phenomenon and in particular the delineation of the effects of various parameters must heavily rely on the experimental observations and measurements.

Divergence of the Stream Surfaces

It has already been shown experimentally[1,2] that the greater the degree of divergence of flow, or the adverse pressure gradient, the less is the swirl that is needed for the vortex core to break down. Thus, ordinarily, increases in the adverse pressure gradient, the circulation imparted to the flow, or the divergence of flow precipitate a breakdown further upstream. As stated by Hall,[3] this is primarily because of the fact "that the pressure gradient along the axis will be larger (more adverse) than the gradients along the outside edge of the vortex if the stream surfaces diverge, by an amount that depends on the square of the circulation." In fact, Hall[4] has demonstrated through numerical experiments with a simple vortex model and the use of the quasi-cylindrical approximation that there is a strong interaction between the different components of flow and a marked response by the breakdown to changes in the surrounding flow.

Randall and Leibovich[5] have shown, through the use of their weakly-nonlinear wave model, that "a stationary wave may occur only if the tube diverges in the direction of flow, i.e., if an adverse pressure gradient exists on the axis."

It would be misleading, however, to suggest that increases in the adverse pressure gradient or the degree of divergence of the tube always moves the breakdown further upstream. In fact, the present work shows that the entire flowfield, in particular the boundary-layer development along the diverging wall, plays important and sometimes unsuspected roles; that it may give rise to flow reversal on the tube wall; and that an increase in the degree of divergence beyond a certain limit does not necessarily lead to the movement of the breakdown further upstream.

The present work examines in detail the effect of the degree of divergence on the type and location of the breakdown in swirling flows in tubes of various angles of divergence, compares the results with those predicted through the use of the analytical models proposed by Randall and Leibovich[5] and by Mager,[6] and illustrates the role played by the flow separation and reversal on the tube wall.

Apparatus

A schematic diagram of the experimental apparatus is shown in Fig. 1. This particular model is considerably longer than that previously used[1,2] and incorporates some refinements as far as the measurement of the flow rate and swirl and the visualization of the flow are concerned.

As before, swirl was imparted to the fluid by 32 streamlined foils placed symmetrically in a circular array around the inlet piece. The simultaneous rotation of the vanes set the desired flux of angular momentum imparted to the flow entering the test tube.

The flow rate was controlled independently of the swirl vanes with a valve and two flow meters placed at the downstream end of the tube. Discharge was varied from 0.05 to 5 ft^3/sec and the vane angle, between the swirl vanes and a radial line, was varied continuously from 0° to 60°.

Four 15-in. long divering tubes were machined to close tolerances to ensure that there would be no obstruction to smooth flow through the bell-mouth, test section, and the outlet tube. The inlet diameter of all test sections was identical to that of the bell-mouth outlet, 1.755 in. The first test section (TS-1) had a half-angle of divergence of $\gamma = 1.38°$, the second 2.36°, the third 3.25°, and the fourth 4.30°. Each test section was installed with an outlet tube that matched the test section exit diameter. The outlet tube was mated with a matching nozzle at its far end to constrict the flow into the 1.50 in. outlet pipe leading to the rotameter bank.

Observations and Measurements

Figures 2–5 show the normalized breakdown position as a function of the Reynolds number for constant circulation. The Reynolds number is defined as $Re = \bar{W}_o D_o / \nu$ where $\bar{W}_o$ is the mean velocity and D_o the diameter at the start of the divergence. The circulation number Ω is defined as $\Omega = \Gamma / \bar{W}_o D_o$ where Γ is the circulation imparted to the flow, i.e., $\Gamma = 2\pi R_i V_i \sin\theta$

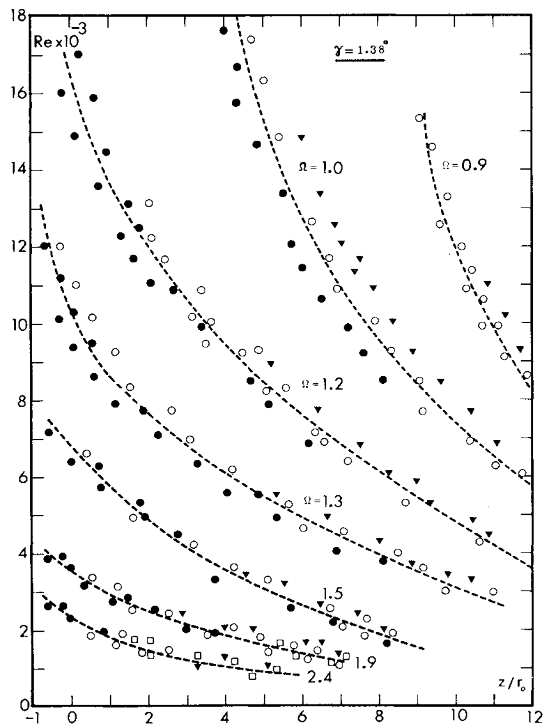

Fig. 2 Vortex-breakdown position as a function of Reynolds and circulation numbers for $\gamma = 1.38°$ (see Fig. 3 for the types of breakdown).

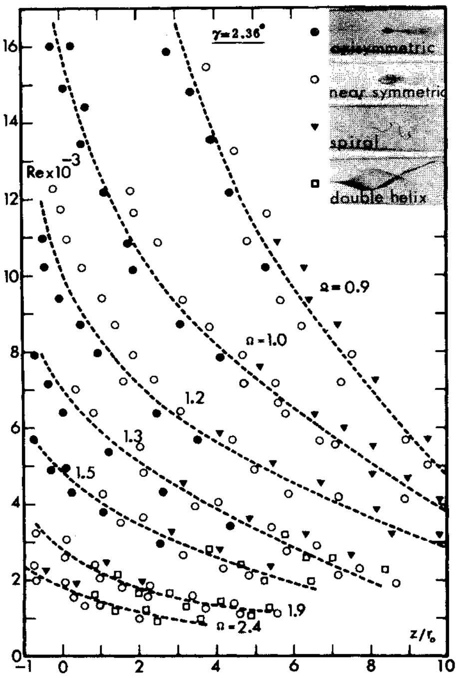

Fig. 3 Vortex-breakdown position as a function of Reynolds and circulation numbers for $\gamma = 2.36°$.

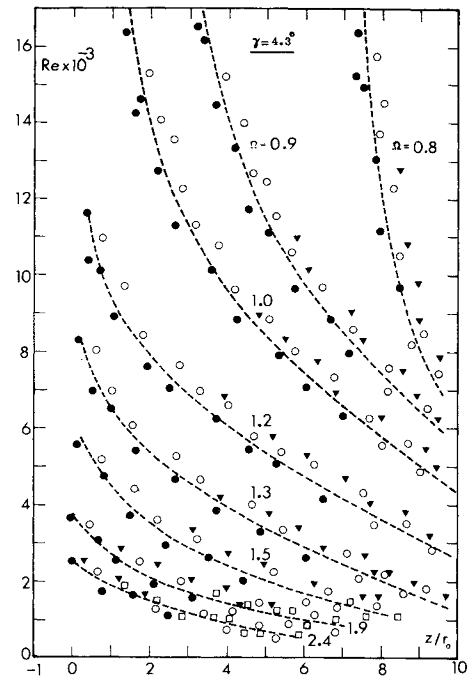

Fig. 5 Vortex-breakdown position as a function of Reynolds and circulation numbers for $\gamma = 4.30°$ (see Fig. 3 for the types of breakdown).

where R_i is the radial distance to the tip of a vane, and V_i is the uniform velocity of flow between any two vanes for a given flow rate and vane-setting angle θ.

Each curve on Figs. 2–5 represents the average of two data runs, weighted to account for the preference for one of the two or three breakdown types which are identified by different symbols.

It is apparent from the data presented that the position of the breakdown is dependent on both the Reynolds and circulation numbers. The dependence on the Reynolds number does not arise from the difficulty of changing the Reynolds number without changing the swirl. This fact has been carefully verified by obtaining two sets of data, i.e., first by maintaining Re constant and changing Ω and then maintaining Ω constant and changing Re. Furthermore, as will be discussed shortly, the theoretical calculations tend to support the fact that the position of the breakdown must depend, among other things, on the Reynolds number.

Although the effect of the divergence of flow or of the adverse pressure gradient on the position of the breakdown may be

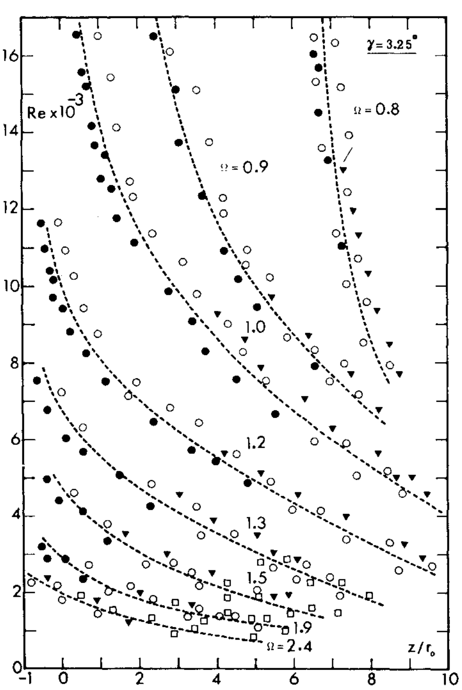

Fig. 4 Vortex-breakdown position as a function of Reynolds and circulation numbers for $\gamma = 3.25°$ (see Fig. 3 for the types of breakdown).

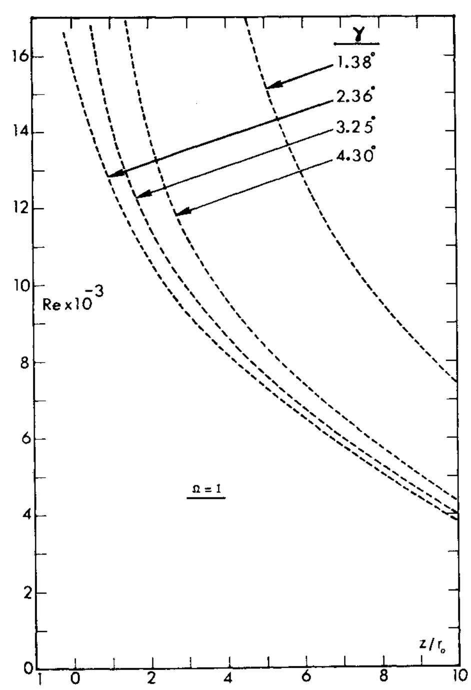

Fig. 6 Vortex-breakdown position as a function of Reynolds number for various angles of divergence and for $\Omega = 1.0$.

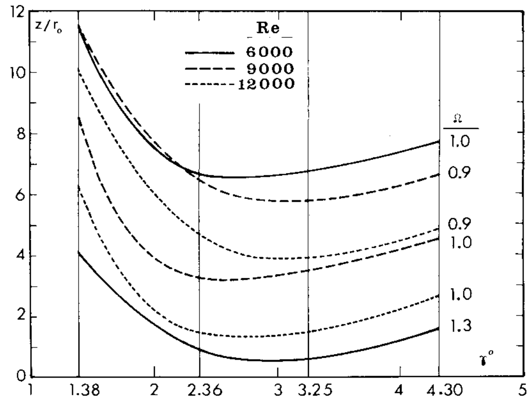

Fig. 7 Vortex-breakdown position as a function of the angle of divergence for various values of Reynolds and circulation numbers. The extension of the curves beyond $\gamma = 2.36$ is valid only for the cases where separation occurs.

deduced from a comparison of the data shown in Figs. 2–5, plots such as shown in Figs. 6 and 7 illustrate the point more emphatically. In Fig. 6, the curves represent the average normalized breakdown position for various Reynolds numbers for each of the four test sections for a single representative circulation number of $\Omega = 1.00$. In Fig. 7, each curve represents the position of the breakdown as a function of the half-angle of divergence for various Re and Ω.

In Fig. 6, the significance of the curves for the first and second test sections is rather obvious, i.e., the data are in conformity with the anticipated conclusion that the retardation of flow along the axis of the tube increases with increasing divergence (more adverse pressure gradient) and the breakdown moves further upstream. However, the meaning of the locations of the curves for the third and fourth test sections is not so obvious. On the basis of the existing reasoning, it would have been expected that the curves for the latter two sections would be far to the left of that for the second test section since the angle of divergence is so much greater. This particular behavior of the breakdown was at first quite surprising. It was only after the outer dye injector was used to visualize the flow pattern in the boundary layer of the tube that the actual fluid action was discovered. Figures 8 and 9 show that the boundary layer separates and reversed flow occurs on the tube wall.

This interaction between the separation on the tube wall and the vortex breakdown points out quite emphatically the fact that there is not a simple set of parameters which would fix the position of the breakdown and that the entire velocity and pressure fields play a very important and interrelated role.

The events depicted by Figs. 8 and 9 may be explained as follows. First, the divergence of the tube decreases the magnitude of the actual pressure gradient along the wall (deceleration of the flow due to the divergence of the tube) and may even cause an adverse pressure gradient if the angle of divergence of the tube is

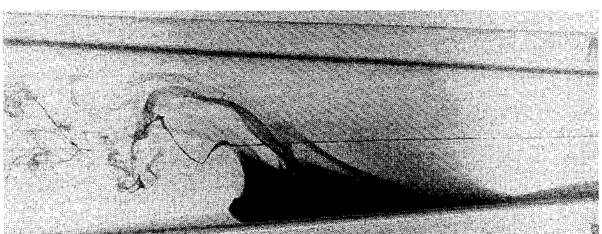

Fig. 8 Vortex breakdown and flow separation ($Re = 3000$, $\Omega = 1.30$, $\gamma = 3.25°$).

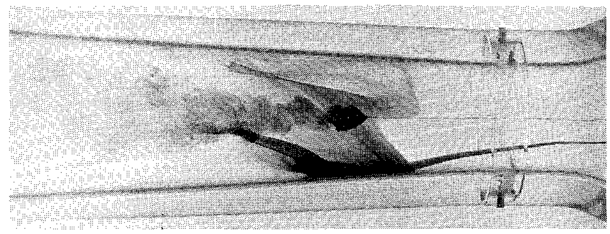

Fig. 9 Vortex breakdown and flow separation ($Re = 10{,}500$, $\Omega = 1.00$, $\gamma = 3.25°$).

large enough provided that the flow remains attached. When the divergence is large enough, however, the flow separates regardless of whether the conditions are just right for the breakdown to occur. Second, the experiments with vortex breakdown have shown[1,7] that in the absence of separation the normalized excess pressure increases slowly as one approaches the breakdown from upstream, reaches a local maximum, then decreases rapidly through the breakdown region, and then slowly increases in the downstream region. Thus, the adverse pressure gradient caused by the breakdown may precipitate the separation of the flow on the wall if it has not already separated. If the flow is separated upstream of the breakdown, i.e., in a region where the flow has not yet come down to a near-critical state from a supercritical state, then the actual adverse pressure gradient seen by the vortex core is considerably less than that which would have existed had there been no separation.

The decrease of the adverse pressure gradient on the wall decreases the same along the core, thereby decreasing the tendency for the occurrence of the breakdown. Furthermore, the relative increase of the axial velocity in the core combined with the above effect pushes the location of the breakdown further downstream. In all our observations of test sections 3 and 4, the boundary-layer separation occurred ahead of the breakdown. Suffice it to say that the foregoing explanation of the observation, that the increase of the tube divergence does not necessarily lead to the movement of the breakdown further upstream, is a highly simplified one. The heretofore unnoted interplay between the adverse pressure gradient caused by the tube divergence, separation of the swirling flow on the tube wall, and the occurrence of the breakdown is considerably more complex. Be that as it may, two facts emerge from these observations. First, the boundary-layer or viscous effects on vortex breakdown in tubes may become very significant. Second, a better simulation of the vortex breakdown is not likely to emerge, particularly because of the circumstances described herein, from solving numerically the full Navier-Stokes equations. Obviously, this will not be possible for a separated flow even if the problem of numerical instability were to be resolved.

Analysis of the Breakdown Location

An attempt has been made to predict the location of the breakdown through the use of two widely different approximate methods devised by Randall and Leibovich[5] and Mager.[6]

Randall and Leibovich[5] presented a model centered about a theory of long, weakly-nonlinear waves propagating on critical flows in tubes of variable cross section. They have calculated, among other characteristics, the location of the breakdown for a tube 1.5 in. in diameter upstream, 2 in. in diameter downstream, and 10 in. long, i.e., with a half-angle of divergence of $\gamma = 1.416°$. Their results, compared with those obtained experimentally by Sarpkaya,[1] were quite encouraging. Thus, it was natural to apply their analysis to the present case. For this purpose the equation [their Eq. (10)] expressing the wave equilibrium position was computerized and solved for $\Omega = 1.36$ (the only circulation number for which Randall and Leibovich's analysis yields a critical flow for the experimentally-obtained

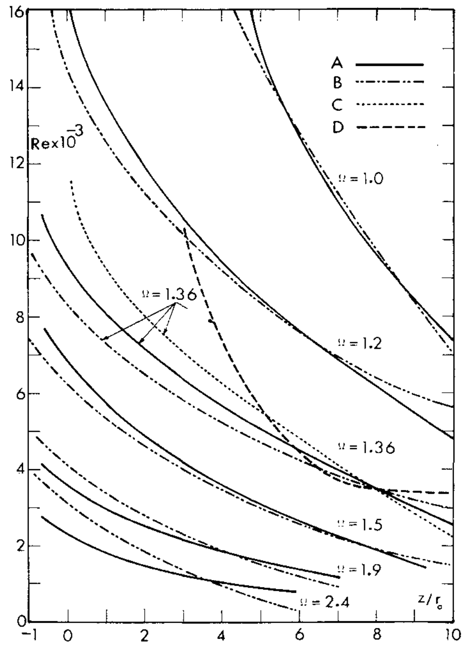

Fig. 10 Comparison of the analytically-predicted breakdown positions with those obtained experimentally: A) experimental; B) analytical via modified version of Mager's method; C) analytical via Randall and Leibovich's method and through the use of Eq. (1); D) analytical via Randall and Leibovich's method and through the use of Eq. (2).

circulation distribution). The tube profile was first expressed by

$$r_b/r_o = 1+(z/r_o)\tan\gamma \tag{1}$$

where r_b is the radius of the bounding wall at a distance z from the start of the diverging wall ($0 < z < 15$ in.), r_o the minimum tube radius (0.8775 in.), and γ is the half-angle of divergence.

Subsequently, the tube profile was expressed by

$$(r_b/r_o)^2 = 1+m[1+\tanh 0.54(z/r_o-8.55)] \tag{2}$$

in which m depends on the angle of divergence and is equal to 0.47, 0.92, 1.35, and 2.0 for test sections 1–4, respectively. The reason for trying this relatively approximate profile is that Randall and Leibovich[5] used a similar expression in their calculations rather than the exact profile given by Eq. (1). The results obtained through the use of the tube profiles given by Eqs. (1) and (2) are shown in Fig. 10 for $\Omega = 1.36$. It is clear that the calculated position of the breakdown is very sensitive to the wall shape chosen even though the viscous terms and hence the boundary-layer development have been neglected. There is not, at present, sufficient experimental evidence to support or refute the predictions of the analysis regarding the sensitivity of the breakdown position to the wall shape. Suffice it to note that the length of the diverging test section plays a significant role for L/r_o ratios less than a certain minimum (approximately 15). In fact, for identical Reynolds and circulation numbers (in particular for high Re and low Ω values) the breakdown in a shorter tube occurs further downstream relative to that in a longer tube. Apparently, the acceleration of the flow in the transition length from a diverging to a uniform tube moves the breakdown downstream as would be anticipated analytically.

The second method used in predicting the breakdown position was a modified version of that developed by Mager.[6,8] The equations of motion for axially symmetric, incompressible flow, with gradients in the axial direction much smaller than those in the radial direction, were reduced by Mager[6] to

$$I_{12}' = 2/Re \tag{3}$$

$$I_{11}'+\tfrac{1}{2}I_{22}'+I_1 W'-(S/2)^2(a'/a) = 0 \tag{4}$$

where

$$I_1 \equiv \int_o^\delta (W-w)r\,dr \tag{5a}$$

$$I_{11} \equiv \int_o^\delta w(W-w)r\,dr \tag{5b}$$

$$I_{12} \equiv \int_o^\delta w\left[1-\left(\frac{rv}{S}\right)\right]r\,dr \tag{5c}$$

$$I_{22} \equiv \int_o^\delta v^2 r\,dr \tag{5d}$$

and

$$P_o' = 2(w_o w_o' - WW')+(S/a)^2 a' - (\Delta p)' = (4/Re)(w_{rr})_o \tag{6}$$

with

$$\Delta p \equiv p_\delta - p_o = 2\int_o^\delta \frac{v^2}{r}\,dr \tag{7}$$

where $a = \delta^2$ and $S = \Gamma/W_o r_o$ are the normalized core area and circulation, respectively. Similarly, all velocities were normalized by the maximum axial velocity W_o, all pressures by $0.5\rho W_o^2$, and all lengths by the radius r_o.

The foregoing equations may be integrated further only if one assumes a set of suitable velocity profiles. One may do so by setting

$$w = W[\alpha+(1-\alpha)f_1] \tag{8a}$$

$$v = V[f_2+\beta f_3], \qquad V = S/\delta \tag{8b}$$

where

$$f_1 = \eta^2(6-8\eta+3\eta^2) \tag{9a}$$

$$f_2 = \eta(2-\eta^2) \tag{9b}$$

$$f_3 = \eta(1-\eta)^2 \tag{9c}$$

with $\eta = r/\delta$.

These velocity profiles satisfy the boundary conditions at $r = 0$ and $r = \delta$, and reduce to the swirling-potential-flow velocity distribution outside the vortex core.† The parameters α and β are initially unknown and must be chosen suitably to represent the initial axial and tangential components of velocity. Finally, to complete the formulation of the analysis, one needs to express the conservation of mass flow within the diverging test section by writing

$$M = WA-2I_1 \tag{10}$$

where A is given by

$$A = [1+(z/r_o)\tan\gamma]^2 \tag{11}$$

Evaluating the integrals given by Eq. (5) through the use of the velocity profiles given by Eq. (8) and inserting the various I_{ij} values in Eqs. (3–7), one obtains the following four equations:

$$[w^2(K_{10}+\alpha K_{11}+\alpha^2 K_{12})-\Gamma^2/4a]a'+[aW^2(K_{11}+2\alpha K_{12})]\alpha'+ [(1/2)\Gamma^2(b_{11}+2\beta b_{12})]\beta'+[2aW(K_{10}+\alpha K_{11}+\alpha^2 K_{12})+ 0.1aW(1-\alpha)]W' = 0 \tag{12a}$$

$$[-0.2W(1-\alpha)]a'+[0.2aW]\alpha'+[A-0.2a(1-\alpha)]W' = -WA' \tag{12b}$$

$$[(\Gamma/a)^2(1+b_{20}+\beta b_{21}+\beta^2 b_{22})]a'+[2\alpha W^2]\alpha'+ [-(b_{21}+2\beta b_{22})\Gamma^2/a]\beta'+[2\alpha^2 W-2W]W' = 48(1-\alpha)/(aRe) \tag{12c}$$

$$[W(K_{20}+\alpha K_{21})]a'+[aWK_{21}]\alpha'+[aW(b_{31}+\alpha b_{41})]\beta'+ [a(K_{20}+\alpha K_{21})] = 2/Re \tag{12d}$$

The coefficients K_{ij} and b_{ij} are tabulated by Mager[8] and will not be reproduced here.

The numerical solutions of the previous equations were carried out by a computer using a differential equations subroutine. This subroutine employs a matrix inversion and the Kutta-Runge integration scheme to solve a system of ordinary differential equations of the first order. The numerical solution was taken to fail or to indicate the location of the breakdown

† A three parameter velocity profile provides additional flexibility in the analysis as far as the slope of the profile at the axis is concerned. This, however, introduces an additional floating parameter the effect of which cannot be evaluated in view of the other approximations made.

when there was such a rapid reduction in the axial velocity and the determinant of the matrix that there appeared to be a limit downstream of which the computation could not proceed.

The results obtained for the flow in the first test section are shown in Fig. 10. The apparent proximity of the experimental and calculated results is somewhat fortuitous because of the fact that the calculated results are dependent on the α and β values selected. For this reason two experimental points have been chosen on one of the experimentally-obtained curves (on $\Omega = 1.00$ curve) and the corresponding α and β values have been determined by trial and error as $\alpha = 1.156$ and $\beta = 2.265$. These values were then used for all other values of Re and Ω. The calculation was then performed for the flow in the second test tube. The results (not shown here) were equally close to those obtained experimentally.

It appears from the foregoing attempts to calculate the vortex-breakdown position that Randall and Leibovich's[5] method is very sensitive to the tube profile and applicable only for one Ω value based on experimentally determined circulation profile. Mager's integral method[6,8] used with a two-parameter velocity profile, on the other hand, is quite sensitive to the shape of the initial profiles. Finally, neither method accounts for the developing boundary layer along the wall even though one could modify and somewhat improve both analyses through the use of an approximate displacement thickness.

Conclusions

The data presented herein show that the adverse pressure gradient resulting from the expansion of an axisymmetric tube has a significant effect on the position of the vortex breakdown when it does occur. An increase in adverse pressure gradient has the same effect on breakdown position as an increase in circulation or the mean-flow rate—that of shifting the breakdown location upstream—as long as the boundary layer does not separate.

An increase in the angle of divergence of the tube beyond some specific value does not result in an increase in the adverse pressure gradient impressed on the vortex core. Instead, the boundary layer separates and reversed flow occurs on the tube wall. This thicker boundary layer, viz. the test section wall, then defines the outer extremity of the potential core, and thereby limits the effective adverse pressure gradient acting on the vortex core. Apparently, the nonlinear interaction between the adverse pressure gradient, separation of the swirling flow, and the vortex breakdown is considerably more complex than that previously anticipated. Furthermore, it does not appear that under the circumstances described herein recourse can be made to the full equations of motion for the prediction of the breakdown characteristics.

The present extension of the analytical model proposed by Mager[6] predicts fairly satisfactorily the location of the breakdown. Furthermore, except for separation, the wall boundary layers do not appear to affect the vortex breakdown since the analytical method neglects the effect of the wall boundary layers. The method proposed by Randall and Leibovich[5] indicates only one value of the normalized circulation ($= 1.36$) for which vortex breakdown will occur. The fact that the vortex breakdown has been shown experimentally to exist also for other values of the normalized circulation tends to refute the validity of that method. Finally, the good agreement obtained with the modified version of Mager's[6] method provides additional experimental evidence that strongly supports Hall's[4] idea that the failure of the quasi-cylindrical equation due to the large axial gradients of the flow indicates the occurrence of the vortex breakdown.

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